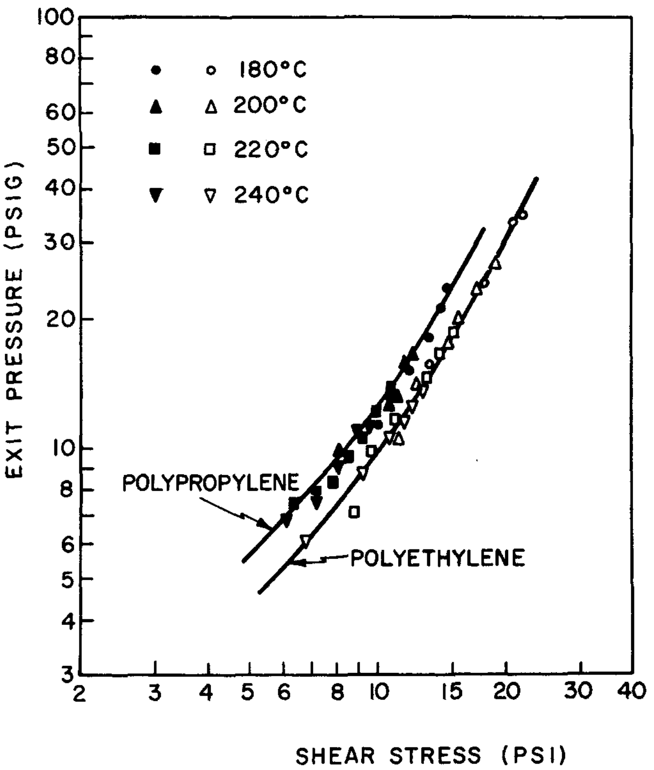

Fig. 3. Exit pressure vs. shear stress for high density polyethylene and polypropylene, respectively, at temperatures 180°, 200°, 220°, and 240°C.

ethylene and polypropylene melts, at four different temperatures. A closer examination of the plot in Figure 3 yields some very interesting observations: the exit pressures of polypropylene melts are consistently higher than those of polyethylene melts in the whole range of shear stresses investigated, and the ratio of the polypropylene exit pressures to the polyethylene exit pressures is about 1.28, which is independent of shear stress and melt temperature as well. Since the exit pressure is approximately equal to the primary normal stress difference (*1, 2*), at least in the case of polymer melts, one might say from the results in Figure 3 that the polypropylene melt is about 28% more elastic than the polyethylene melt.

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Experimental Measurements of Loss Coefficients in the Entrance Region of a Pipe for Viscous Power Law and Viscoelastic Fluids

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When a viscous fluid enters a pipe through an abrupt contraction, its velocity profile undergoes a change from its initial entrance form, usually assumed to be flat, to that of a fully developed profile at an axial position far downstream. The development of the boundary layer in the entrance region causes the acceleration of the central core of the fluid, and the high velocity gradient at the pipe wall results in a greater viscous dissipation than in the fully developed region. The losses in the entrance region are usually reported as a loss coefficient which is the sum of the increase in kinetic energy of the fluid and the increase in viscous frictional losses in excess of the losses that would have existed for fully developed flow. This loss coefficient, expressed as an equivalent number of velocity heads, is a function of the axial distance from the tube entrance and reaches an asymptotic value as the flow becomes fully developed.

Application of the mechanical energy balance for a viscous power law fluid between a station 1, immediately downstream of the contraction where the velocity profile is assumed to be flat, and a station 2, in the fully developed region (Figure 1) yields

$$\frac{p_1 - p_2}{\frac{\rho V^2}{2}} = \frac{32\, X/R}{N_{Re'}} + C \qquad (1)$$

where

$$N_{Re'} = \frac{D^n\, V^{2-n}\, \rho}{k\, 8^{n-1} \left(\frac{3n+1}{4n} \right)^n} \qquad (2)$$

$$C = \frac{\Delta p_{EX}}{\frac{\rho V^2}{2}} + \frac{3(3n+1)^2}{(2n+1)(5n+3)} - 1 \qquad (3)$$

and Δp_{EX} is the frictional loss over and above the fully developed loss. Schmidt and Zelden (*1*) have summarized the theoretical and experimental values of C obtained by different investigators for Newtonian fluids. Astarita and Greco (*2*) have suggested a value of $C = 4.48$ for a Newtonian fluid in the entrance of a pipe from an abrupt contraction. This value is far greater than any of the previously published values which range from 1.0 to 1.41.

Theoretical values for C as a function of n for power law fluids have been presented by Bogue (*3*) and Collins and Schowalter (*4*). Both analyses assumed a flat entry profile in solution of the boundary-layer equations, and both compared their pressure drop predictions with the data of Dodge (*4*). Good agreement was obtained, yet the available experimental data for C as a function of n as summarized by Skelland (*5*) do not agree with either the predicted results of Bogue or of Collins and Schowalter.

For a viscoelastic fluid, the influence of the deviatoric normal stresses, particularly the components in the direction of flow, must be considered. Philippoff and Gaskins (*6*) suggested that an additional term is required in the mechanical energy balance to account for the elastic energy imparted to a fluid at the entrance of a pipe. This elastic energy was taken to be proportional to the recoverable shear. The capillary rheometer data of Bagley (*7*), Bauer and Weber (*8*), and Tordella (*9*) were used to substantiate the form of the mechanical energy balance including the elastic energy term. These results, for low Reynolds numbers, indicated that the excess losses as a result of boundary-layer development in the capillary are very small compared with the fully developed losses in the capillary and the elastic and viscous losses upstream of the capillary entrance.

Astarita, Greco, and Peluso (*10*) pointed out that in addition to a term in the mechanical energy balance to account for the increase in elastic energy of a fluid as it enters a pipe from an abrupt contraction, it is also necessary to account for the work done by elastic forces on the fluid as it enters and leaves the control volume shown in Figure 1. Experimental data for viscoelastic fluids flowing through an abrupt contraction were also presented by Astarita, Greco, and Peluso. It was concluded that their data contradicted a previous hypothesis (*11*), which stated that the hydrostatic pressure drop through an abrupt contraction is larger in the case of a viscoelastic liquid than for a purely viscous liquid, and that if there is any significant effect of elasticity, it appears to be in the opposite direction. However, their experimental results were not conclusive, and sufficiently high Reynolds numbers were not obtained to allow for experimental determination of the loss coefficients C for a viscoelastic fluid. No experimental value of the loss coefficient in the entry region of a pipe appear to be available for viscoelastic fluids.

EXPERIMENTAL

The flow system, shown schematically in Figure 2, consisted of a 400-gal. tank fitted with a steam coil, a 15-hp. monopump, a surge tank, and the test section. Fluid flowed through a 2-in. I.D. pipe from the pump to a 20 ft. long, 1-in. I.D. copper test pipe, via a sharp edged contraction. Pressure tappings were accurately located at 2 ft. intervals along the test section, the first tapping being within 1 diam. of the sharp edged contraction. Each pressure tapping was manifolded to a mercury manometer. The return line from the test section passed into a 44-gal. drum mounted on a 550 lb. capacity platform scale. Copper constantan thermocouples were located at positions in the flow system as shown in Figure 2.

When steady state was reached, the pressure drops and system temperatures were recorded. Mass flow rates were determined by measuring the time required to accumulate 100 lb. of fluid.

The rheological properties of the fluids were determined by using a 7.5-cm. cone and plate combination on an R 16 Weissenberg rheogoniometer. The total force technique was used for measurement of normal stresses (*12*). Evaporation effects during normal stress measurements were eliminated by using the technique already described (*13*). No corrections to the normal stress data were made for centrifugal force or surface tension effects as outlined by Ginn and Metzner (*14*).

The test fluids employed were aqueous solutions of methocel and separan AP 30. Samples were collected before and after each run on the flow system. Shear and normal stress measurements as a function of shear rate were made on each sample at the temperature of the run. No shear stress degradation was noticed for either of the samples during a run. A thixotropy in the normal stress behavior was observed for the separan solutions. This is discussed later. No permanent normal stress degradation for methocel was observed during a run on the flow system. High concentration of both solutions showed permanent degradation in shear and normal stress after long periods of storage, presumably due to enzymatic attack.

RESULTS AND CONCLUSIONS

Table 1 lists the power law constants for all the fluids investigated, the temperature at which the measurements were made, and the shear rate range over which the results are valid. Also listed are those fluids which exhibited a measurable normal stress. M is used to designate a methocel solution and S for a separan solution.

A plot of the friction factor based on pressure loss measurements in the fully developed region of the test section vs. the generalized Reynolds number is shown in Figure 3. The generalized Reynolds number was calculated from Equation (2) by using n and k listed in Table 1 and the measured mass flow rate. These results are in excellent agreement with $f = 16/N_{Re'}$, thus substantiating the experimental technique employed.

VISCOUS FLUIDS

On Figure 4 is shown a typical plot of $(p_1 - p_2)/(\rho V^2/2)$ vs. $(X/R)/N_{Re'}$ for the purely viscous methocel solution M4. By drawing a line of slope 32 through the linear portion of Figure 4 and by extrapolating to zero on the abscissa, the value of C in Equation (1) was determined for solution M4. The values of C obtained in this manner are plotted as a function of n and compared with the theoretical predictions of Bogue and of Collins and Schowalter in Figure 5. The purely viscous methocel solutions show an average deviation of $+4\%$ from Bogue's theoretical predictions and -10% from those of Collins and Schowalter.

The behavior of separan AP 30 solutions proved to be rather interesting. Although these solutions exhibited a very high first normal stress difference upon initial shearing, it was found that $P_{11} - P_{22}$ for the three solutions tested in the flow system decayed to zero after about 1½ hr. of shearing in the rheogoniometer. This behavior is

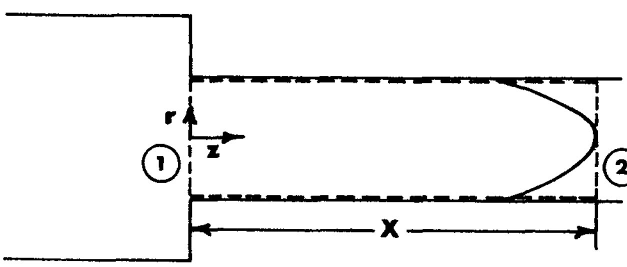

Fig. 1. Control volume for flow after a sudden contraction.

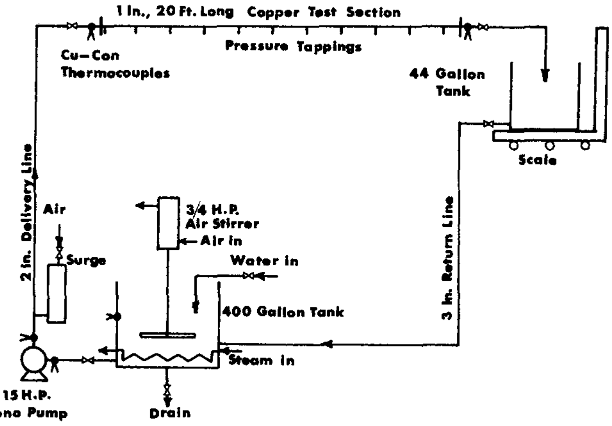

Fig. 2. Schematic diagram of flow system.

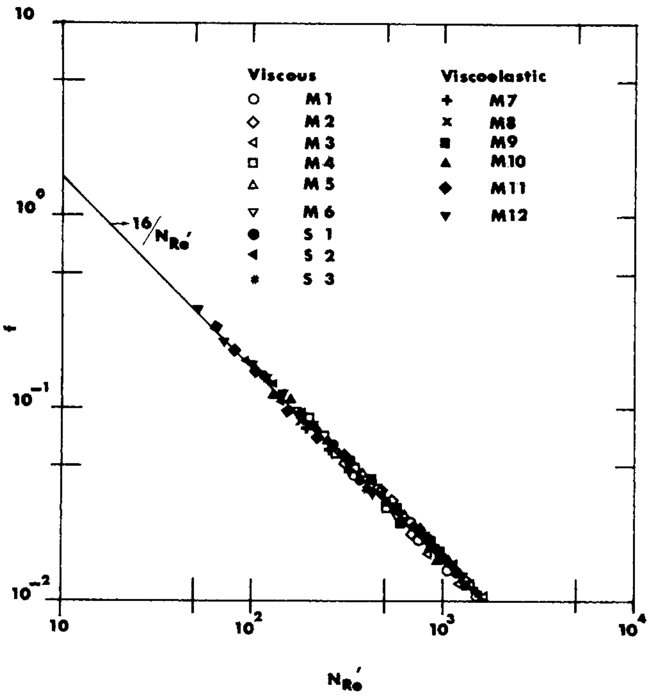

Fig. 3. Friction factor vs. generalized Reynolds number.

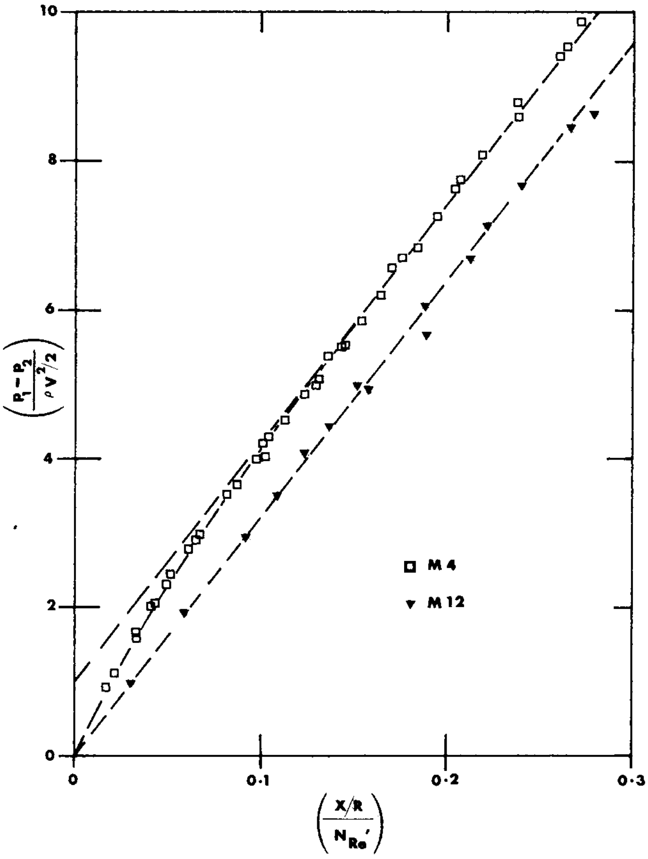

Fig 4. Experimental axial pressure distribution for a purely viscous methocel solution and a viscoelastic methocel solution.

shown in Figure 6, where $P_{11} - P_{22}$ is plotted as a function of time at a shear rate of 232 sec.$^{-1}$. This shear rate is representative of those encountered in the flow system. Notice also from Figure 6 that the sample had not as yet returned to its initial state after a 2 min. rest period. Since each run on the flow system was carried out over a period of about 8 hr., during which time the fluid was being continually sheared, it is not surprising after a look at Figure 6 that the separan solutions showed no deviations from purely viscous behavior. The experimental values of C for the separan solutions are shown in Figure 5. The average deviations between the experimental values and the Collins and Schowalter theory is + 1.5%.

Astarita and Greco (2) have suggested that the velocity profile for purely viscous fluids is not uniform at the entrance of an abrupt contraction as was assumed in the analyses of Bogue and of Collins and Schowalter. The results of this work for a 2 to 1 in. contraction indicate that for $0.365 \leq n \leq 0.858$ and for $200 \leq N_{Re'} \leq 2{,}000$ a flat profile can be assumed, and either the results of Bogue or of Collins and Schowalter can be used to predict the losses in the tube entrance from a sudden contraction.

TABLE 1. FLOW PROPERTIES OF THE FLUIDS INVESTIGATED

Fluid	Temp.	Shear rate range, sec.$^{-1}$	n	$k \times 10^3$ lb$_f$. ft.$^{-2}$ sec.^{n}	Measurable normal stress
M1	20	100 to 1,400	0.858	0.50	No
M2	20	100 to 1,400	0.838	1.88	No
M3	20	100 to 1,400	0.752	1.90	No
M4	20	100 to 1,400	0.724	4.31	No
M5	20	100 to 1,400	0.645	9.0	No
M6	20	100 to 1,400	0.580	22.6	No
M7	20	100 to 1,000	0.541	33.5	Yes
M8	20	100 to 1,000	0.538	25.0	Yes
M9	20	100 to 1,000	0.516	73.2	Yes
M10	40	100 to 1,000	0.470	62.6	Yes
M11	20	100 to 1,000	0.465	152	Yes
M12	20	100 to 1,000	0.400	230	Yes
S1	20	6 to 1,400	0.365	83.6	No
S2	20	6 to 1,400	0.372	62.6	No
S3	20	6 to 1,400	0.398	47.7	No

However, for design purposes, the results of Collins and Schowalter are recommended because of the greater range of applicability and apparent conservative nature of their predictions, particularly at high values of n.

VISCOELASTIC FLUIDS

The flow behavior index n and the consistency factor k for the viscoelastic methocel solutions examined are listed in Table 1. Figure 7 shows the first normal stress difference, $P_{11} - P_{22}$, as a function of shear rate, assuming the equality of P_{22} and P_{33}. The loss coefficients were determined from a plot of $(p_1 - p_2)/(\rho V^2/2)$ vs. $(X/R)/(N_{Re'})$ by using the method previously discussed. A typical result for solution M12 is shown in Figure 4. The experimental loss coefficients C for the viscoelastic methocel solutions when $50 \leq N_{Re'} \leq 1{,}742$ are compared to the viscous theories in Figure 5. Comparison of the first normal stress difference to the experimentally determined loss coefficient, for corresponding solutions, clearly shows that as the normal stress difference increases, the loss coefficient decreases from that predicted from the viscous theories until the coefficient approaches zero. The results for solutions M9, M11, and M12, where the loss coefficient is zero, indicate that the velocity profile is fully developed at the entrance of the pipe.

From the gradual decrease in loss coefficient from the purely viscous value to $C = 0$ for highly elastic methocel solutions, as is shown in Figure 5, one might conclude that there is a gradual transition from a flat velocity profile at the pipe entrance for purely viscous solutions to a fully developed profile as the elasticity of the methocel solutions increases. For highly elastic methocel solutions such as M9, M11, and M12, it appears as if the profile is developing in anticipation of the contraction and that the flow

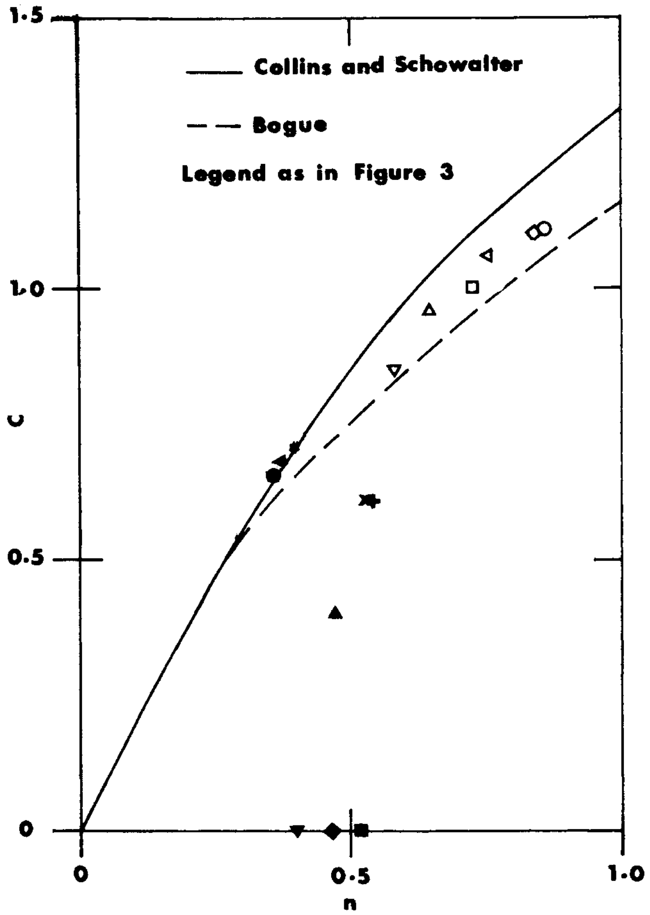

Fig. 5. Loss coefficient as a function of flow behavior index.

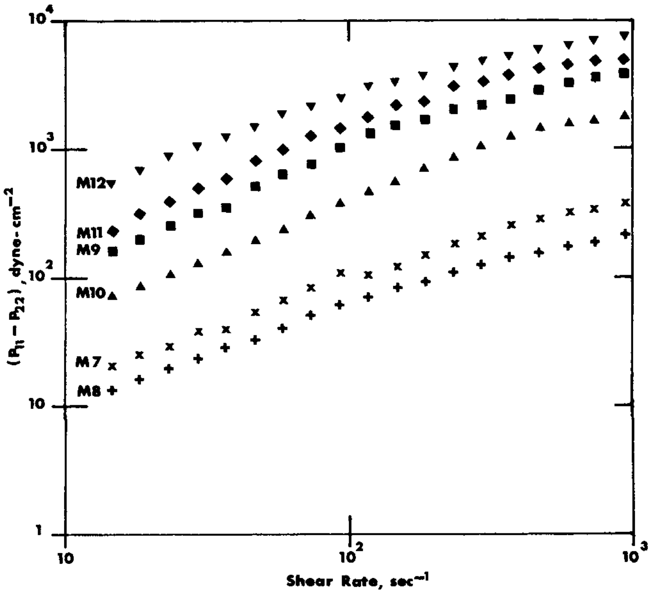

Fig. 7. The first normal stress difference as a function of shear rate for the methocel solutions tested.

behavior of these fluids upstream of the contraction is of paramount importance.

It must be emphasized that results for viscoelastic fluids were obtained for aqueous solutions of one polymer on the downstream side of a 2 to 1 contraction when $50 \leq N_{Re'} \leq 1{,}742$ in the 1-in. pipe. Generalizations to other viscoelastic fluids and contraction ratios cannot be made without further experimental data.

ACKNOWLEDGMENT

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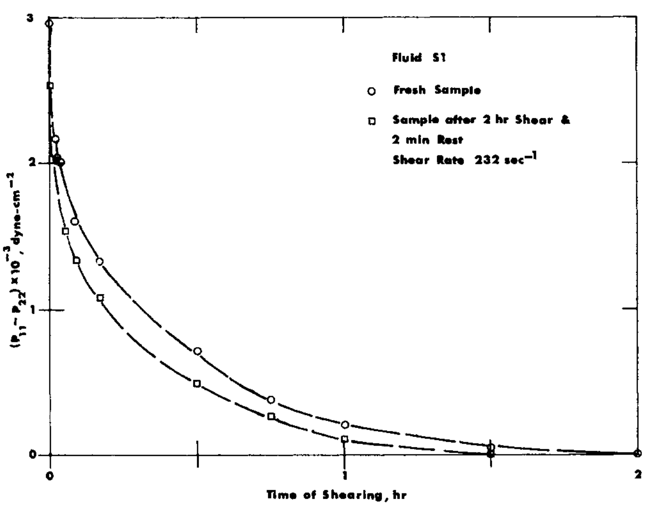

Fig. 6. Thixotropy in the normal stress behavior for a separan AP 30 solution.

NOTATION

C = loss coefficient
k = consistency factor in power law
n = flow behavior index in power law
$N_{Re'}$ = Reynolds number for power law fluid
P_{11}, P_{22}, P_{33} = deviatoric normal stress in the z, r, and θ directions
P = hydrostatic pressure
r, θ, z = cylindrical coordinates
R = radius of pipe
v_z = local velocity in the z direction
V = average velocity
X = distance downstream of contraction
ρ = density

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